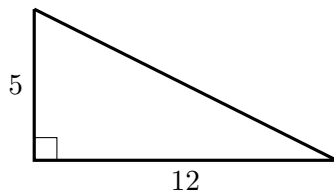


NMC SAMPLE PROBLEMS: GRADE 9

1. One root of the cubic polynomial $x^3 + 2x^2 - 4x + 1$ is 1. What is the sum of the other two roots of this polynomial?
(a) 1 (b) -1 (c) 3 (d) -3 (e) None of these
2. A pouch contains two red balls, three blue balls and one green ball. What is the probability of drawing a red ball and the green ball if two balls are drawn randomly?
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{6}$ (d) $\frac{2}{15}$ (e) None of these
3. A line of slope 2 passes through the points (2, 4) and (3, b). Find the value b .
(a) 8 (b) 6 (c) 4 (d) 2 (e) None of these
4. Find the solution of the system of equations $\begin{cases} 3x + 6y = -3 \\ 2x + y = -8 \end{cases}$.
(a) (5, -2) (b) (-5, 2) (c) (2, -5) (d) (-2, -5) (e) None of these
5. If $f(x) = 3x^2$, what is $f(f(x))$?
(a) $3x^4$ (b) $9x^2$ (c) $9x^4$ (d) $27x^4$ (e) None of these
6. There is an isosceles triangle which is also a right triangle. Its area is 50 square inches. Find the length of the base of the triangle.
(a) $2\sqrt{5}$ inches (b) 5 inches (c) $5\sqrt{2}$ inches (d) $4\sqrt{10}$ inches (e) None of these
7. Suppose that a, b, c, d are positive integers satisfying $\frac{3a}{b} = \frac{2c}{d}$. Which of the following is not necessarily true?
(a) $a : b = 2c : 3d$ (b) $3a : 2b = c : d$ (c) $3ad = 2bc$ (d) $\frac{3a+2b}{b} = \frac{2c+2d}{d}$
(e) None of the above
8. What is the twenty-first term in the arithmetic sequence 2, 9, 16, 23, ...?
(a) 142 (b) 149 (c) 156 (d) 163 (e) None of these
9. What is the sum of the first 21 terms of the arithmetic sequence 2, 9, 16, 23, ...?
(a) 1511 (b) 1660 (c) 1816 (d) 1979 (e) None of these

10. Solve the equation $|x + 4| = 3 - 2x$ for x .
- (a) $7, -\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 7 (d) $7, \frac{1}{3}$ (e) None of these
11. From a group of twenty people, how many ways can the officers: President, Vice-president, Secretary, and Treasurer be chosen?
- (a) 4 (b) 24 (c) ${}_{20}C_4$ (d) ${}_{20}P_4$ (e) None of these
12. (Refer to Problem #11) Now they have chosen the four officers. The President wants two officers and two non-officers to join him on a walk. How many ways are there to choose his four fellow walkers?
- (a) 360 (b) 720 (c) 1440 (d) 2880 (e) None of these
13. A ball is thrown up into the air from a patio in the two-story building which is at 30 feet high. The ball will be h feet above the ground at t seconds, and $h = 30 + 128t - 16t^2$. Find the maximum height in feet which the ball can reach.
- (a) 286 (b) 316 (c) 226 (d) 512 (e) None of these
14. A square is inscribed in a circle of radius r . What fraction of the area of the circle lies outside the square?
- (a) $\frac{r}{\pi}$ (b) $\frac{\pi-2}{\pi}$ (c) $\frac{\pi r}{\pi-\sqrt{2}}$ (d) $\frac{\pi r}{2}$ (e) None of these
15. A newly discovered element Yourelementium, has a half-life of 8 months. How much of a 20 g sample of this new element will remain after 24 months? (Note: The half-life of a chemical substance is the time required for half of a given substance to disintegrate.)
- (a) 0 g (b) 5 g (c) 2.5 g (d) 1.25 g (e) None of these
16. If you convert 315_9 to Base Ten, what number do you have?
- (a) 72 (b) 257 (c) 380 (d) 729 (e) None of these
17. A right triangle has two legs of length 5 and 12. What is the length of the altitude to the hypotenuse? (Note: Figure not drawn to scale!)



- (a) $\frac{2}{5}$ (b) $\frac{60}{13}$ (c) $\frac{65}{12}$ (d) $\frac{12\sqrt{2}}{5}$ (e) None of these

18. How many diagonals does a nonagon (a nine-sided polygon) have?
(a) 44 (b) 36 (c) 27 (d) 14 (e) None of these
19. Farmer Bob has 400 ft of fencing to enclose a rectangular field. What is the maximum possible area (in ft^2) that he can enclose?
(a) 10000 (b) 16000 (c) 20000 (d) 40000 (e) None of these
20. Let the three roots of the equation $x^3 - 3x^2 - 4x - 7 = 0$ be a , b and c . What is $a^2b^2c + a^2bc^2 + ab^2c^2$?
(a) 12 (b) -12 (c) -21 (d) -28 (e) None of these
21. If Alice's *ipad* crashes, she will be angry. Which of the following statements is also true?
(a) If Alice is angry, then her *ipad* has crashed.
(b) If Alice is not angry, then her *ipad* has not crashed.
(c) If Alice's *ipad* does not crash, then she will be angry.
(d) Either Alice is not angry or her *ipad* has crashed.
(e) None of the above
22. What is the sum of the coefficients of the expression $(3a + 2b + c)^4$ when expanded and simplified?
(a) 6 (b) 36 (c) 625 (d) 1296 (e) None of these
23. How many ways can I arrange the six letters of the word LIGHTS such that no vowel occupies an odd-numbered position?
(a) 60 (b) 120 (c) 360 (d) 720 (e) None of these
24. Alice, Bob and Chris must clean their dorm. If it takes Alice 1 hour to clean it alone, Bob 2 hours to clean it alone, and Chris 1.5 hours to clean it alone, how many hours does it take the three of them working together to finish the task?
(a) $\frac{5}{12}$ hours (b) 1.5 hours (c) 0.6 hours (d) 0.5 hours (e) None of these
25. (Refer to Problem #24) Suppose all three begin the task at the same time. However, Alice, Bob and Chris pull the fire alarm 20 minutes after they begin. What fraction of the task remains?
(a) $\frac{6}{13}$ (b) $\frac{7}{13}$ (c) $\frac{7}{18}$ (d) $\frac{5}{18}$ (e) None of these

26. Let $a = \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$. Then a is a real number. Find a .
- (a) $\sqrt{2} + 1$ (b) $\sqrt{2} - 1$ (c) $-1 - \sqrt{2}$ (d) $\frac{2}{5}$ (e) None of these
27. Alice has two bags of marbles: Bag A contains 8 red and 3 blue marbles, and Bag B contains 5 red and 7 blue marbles. She randomly select a bag and then draw one marble at random. It is blue. What is the probability that it came from Bag B ?
- (a) $\frac{7}{12}$ (b) $\frac{7}{10}$ (c) $\frac{77}{113}$
(d) Not enough information (e) None of these
28. How many positive integral divisors does the number 216 have?
- (a) 16 (b) 17 (c) 18 (d) 19 (e) None of these
29. Compute $\phi(2015)$, where $\phi(n)$ is the number of positive integers less than and relatively prime to n .
- (a) 3 (b) 1440 (c) 1608 (d) 2014 (e) None of these
30. How many ordered triples of positive integers (x, y, z) satisfy $x + y + z \leq 7$?
- (a) 11 (b) 18 (c) 24 (d) 44 (e) None of these
31. What is the coefficient of xy^2z in the expansion of $(x + y + z)^4$?
- (a) 3 (b) 4 (c) 6 (d) 12 (e) None of these
32. What is the probability of rolling a sum of 11 with three standard six-sided dice?
- (a) $\frac{11}{216}$ (b) $\frac{1}{8}$ (c) $\frac{25}{216}$ (d) $\frac{7}{72}$ (e) None of these
33. A square with vertices $(0, 0)$, $(2, 0)$, $(0, 2)$ and $(2, 2)$ is rotated around the y -axis. Compute the volume of the resulting figure.
- (a) 4π (b) 8π (c) 12π (d) 16π (e) None of these
34. What is the sum of all prime factors of 2015?
- (a) 49 (b) 50 (c) 2064 (d) 2065 (e) None of these
35. Solve the inequality $|4 - 5x| \leq 8$ for x .
- (a) $-\frac{4}{5} \leq x \leq \frac{12}{5}$ (b) $x < -\frac{4}{5}$ or $x > \frac{12}{5}$ (c) $-\frac{12}{5} \leq x \leq \frac{4}{5}$
(d) $x \geq \frac{4}{5}$ or $x \leq -\frac{12}{5}$ (e) None of these

36. What is the set of points equidistant from $(3, 3)$ and $(6, 0)$.
(a) circle (b) line (c) parabola (d) hyperbola (e) None of these
37. The area of triangle ABC is 18. If the coordinates of A and B are $(-3, 2)$ and $(3, 2)$ respectively, what is the locus of point C ?
(a) $y = 3x + 3$ (b) $y = 8$ or $y = -4$ (c) $y = 3$ or $y = -3$
(d) $x = 6$ or $x = -6$ (e) None of these
38. In New-Kid's-Town, required bicycle licenses consist of two letters followed by three digits. (For example, LV123, AA000, MP906, ZZ999, and so on.) How many distinct license numbers are available?
(a) 676,000 (b) 628,600 (c) 628,340 (d) 300,600 (e) None of these
39. Given that $x - 1$ and $x + 1$ are reciprocal of each other, what is x^2 ?
(a) 2 (b) $\sqrt{2}$ (c) 4 (d) 1 (e) None of these
40. The length of the shortest side of a right triangle is 11. If the lengths of all three sides are integers, what is the perimeter of the triangle?
(a) 241 (b) 198 (c) 132 (d) 121 (e) None of these
41. Consider point $P = (2, 3)$ on the Cartesian plane. If you first reflect P about the line $y = x$ and then rotate it about the origin by 90° counterclockwise, what are the coordinates of the new position of P ?
(a) $(-2, 3)$ (b) $(-2, -3)$ (c) $(2, -3)$ (d) $(3, 2)$ (e) None of these
42. For what real values of x does the expression below represent a real number?
$$\sqrt{\frac{x+4}{2-x}}$$

(a) $x < 2$ (b) $-4 \leq x < 2$ (c) $-3 \leq x \leq 4$ (d) $-3 \leq x$ (e) None of these
43. What is the value of $\frac{(2015 + 2016)^2 - 2015^2 - 2016^2}{2015 \times 2016}$?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

44. Find the area of parallelogram $ACDB$ with vertices

$$A = (0, 0), B = (3, 1), C = (4, 4), D = (7, 5).$$

- (a) 11 (b) 10 (c) 9 (d) 8 (e) None of these
45. Consider functions f whose domain is given by $\{1, 2, 3\}$ and whose values lie in the set of integers $\{0, 1, 2, 3, 4\}$. How many different functions f can be formed?
- (a) 60 (b) 100 (c) 125 (d) 243 (e) None of these
46. A chemist mixes distilled water with a 90% solution of sulfuric acid to produce a 50% solution. If 5 liters of distilled water is used, how much 50% solution is produced? (Answer in liters using decimals.)
- (a) 10.75 (b) 11.25 (c) 12.5 (d) 13.75 (e) None of these
47. Evaluate the sum of the first 21 values $\frac{1}{\sqrt{3n+1}+\sqrt{3n-2}}$ starting from $n = 1$ (up to $n = 21$): that is the value
- $$\frac{1}{\sqrt{4} + \sqrt{1}} + \frac{1}{\sqrt{7} + \sqrt{4}} + \cdots + \frac{1}{\sqrt{64} + \sqrt{61}}.$$
- (a) $\frac{7}{3}$ (b) $\frac{8}{3}$ (c) 3 (d) $\frac{11}{3}$ (e) None of these
48. What is the sum of the measures of the interior angles of a hexagon (a six sided polygon)? (Answer in degrees.)
- (a) 360 (b) 480 (c) 600 (d) 720 (e) None of these
49. A 6-faced die is tossed twice. What is the probability that the same numbers are shown in both toss?
- (a) $\frac{1}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$ (e) None of these
50. An average mountain bald eagle is 28 inches long, weighs 70 ounces and has a wingspan of 100 inches. Assume that wingspan is directly proportional to the fraction $\frac{\text{weight}}{\text{length}}$. If I am a mountain bald eagle that is 2 feet, 6 inches long and weighs 5 pounds and 10 ounces, what is my wingspan in inches? (Hint: 1 foot=12 inches and 1 pound=16 ounces)
- (a) 90 (b) 100 (c) 110 (d) 120 (e) None of these
51. How many ordered pairs (x, y) of positive integers x and y satisfy the equation $3x + 5y = 80$?
52. What is the area of a regular hexagon (a six sided polygon) inscribed in a circle with radius 1? (Answer in fraction.)

53. If a fair coin is tossed 3 times, what is the probability of getting at least 2 heads (“at least 2 heads” means 2 or more heads)?
54. How many ways are there to distribute 9 (indistinguishable) balls into 3 different homerooms if each homeroom must receive at least one ball?
55. In a pizza shop, it is possible to have on a pizza a choice of any combination of the following eight ingredients:
bacon, mushrooms, onion, pepperoni,
peppers, salami, sardines, sausage.
How many different varieties of pizza can be ordered if the pizza shop orders all pizzas with at least three toppings of any combination of the eight ingredients?

▷ KEYS ◁

[1] (d)	[15] (c)	[29] (b)	[43] (b)
[2] (d)	[16] (b)	[30] (e)	[44] (d)
[3] (b)	[17] (b)	[31] (d)	[45] (c)
[4] (b)	[18] (c)	[32] (b)	[46] (b)
[5] (d)	[19] (a)	[33] (b)	[47] (a)
[6] (e)	[20] (d)	[34] (a)	[48] (d)
[7] (e)	[21] (b)	[35] (a)	[49] (c)
[8] (a)	[22] (d)	[36] (b)	[50] (d)
[9] (e)	[23] (c)	[37] (b)	[51] 5 (pairs)
[10] (b)	[24] (e)	[38] (a)	[52] $\frac{3\sqrt{3}}{2}$
[11] (d)	[25] (d)	[39] (a)	[53] $\frac{1}{2}$
[12] (a)	[26] (b)	[40] (c)	[54] 28
[13] (a)	[27] (c)	[41] (a)	[55] 219
[14] (b)	[28] (a)	[42] (b)	