

## NMC SAMPLE PROBLEMS: GRADE 10

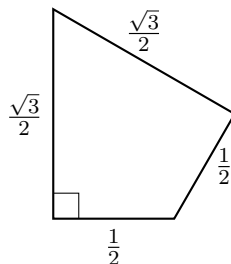
1. Find the remainder when  $P(x) = x^{100} + 2x + 1$  is divided by  $x + 1$ .  
(a) 0                      (b) 1                      (c) 4                      (d) -2                      (e) None of these
  
2. How many positive numbers less than 100 are there that have odd number of positive divisors?  
(a) 25                      (b) 13                      (c) 10                      (d) 9                      (e) None of these
  
3. If 
$$\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = 2018,$$
what is  $\frac{a-b}{a+b}$ ?  
(a)  $\frac{1}{2018}$                       (b)  $-\frac{1}{2018}$                       (c) 2018                      (d) -2018                      (e) None of these
  
4. A palindrome is a number that reads the same backward or forward. For example, 35853 is a palindrome. There is one prime number that is a factor of all palindromes with an even number of digits. What is the prime number?  
(a) 7                      (b) 11                      (c) 13                      (d) 17                      (e) 19
  
5. If both roots of the quadratic equation  $x^2 - 99x + k = 0$  are prime numbers, find the value of  $k$ .  
(a) 194                      (b) 288                      (c) 291                      (d) 470                      (e) None of these
  
6. Simplify  $\sqrt[3]{a^2 \sqrt[4]{a} \sqrt{a}}$ , where  $a$  is a positive real number.  
(a)  $a^{19/24}$                       (b)  $a^{1/2}$                       (c)  $a^{-1/2}$                       (d)  $a^{5/2}$                       (e) None of these
  
7. If  $a$  and  $b$  are two distinct numbers such that  $2a^2 - 7a = 11$  and  $2b^2 - 7b = 11$ , what is  $ab$ ?  
(a)  $\frac{7}{2}$                       (b)  $-\frac{7}{2}$                       (c)  $\frac{11}{2}$                       (d)  $-\frac{11}{2}$                       (e)  $\frac{11}{7}$
  
8. Suppose  $\alpha$  and  $\beta$  are nonzero real solutions of the equation  $x^2 - 6x + 2 = 0$ . Find the sum  $\frac{1}{\alpha} + \frac{1}{\beta}$ .  
(a) 3                      (b)  $\sqrt{2}$                       (c) 7                      (d)  $\sqrt{7}$                       (e) None of these

9. Define  $a\clubsuit b = ab + 3a + 2b$  for all real numbers  $a$  and  $b$ . If a pair of positive numbers  $(x, y)$  satisfies  $x\clubsuit y = y\clubsuit x$  and  $x\clubsuit y = 14$ , what is  $x + y$ ?
- (a) 2                      (b) 4                      (c) 6                      (d) 8                      (e) 10
10. How many arrangements of the letters in *INFINITI* are there such that the consonants occur in alphabetical order?
- (a) 840                      (b) 420                      (c) 280                      (d) 140                      (e) 70
11. Compute the perimeter of the triangle whose vertices are  $(0, 0)$  and two intercepts of the line  $5x + 12y = 60$ .
- (a) 25                      (b) 30                      (c) 35                      (d) 45                      (e) 50
12. Determine  $i + i^2 + i^3 + i^4 + \dots + i^{2018}$ , where  $i^2 = -1$ .
- (a)  $-1 + i$                       (b) 0                      (c)  $-i$                       (d)  $-1 - i$                       (e)  $1 + i$
13. How many consecutive zeros are there at the end of  $100!$ .
- (a) 18                      (b) 100                      (c) 25                      (d) 24                      (e) None of these
14. Find  $\sin(\cos^{-1} x)$  in terms of  $x$ .
- (a)  $\frac{1}{x}$                       (b)  $\frac{1-x}{x}$                       (c)  $\sqrt{1-x^2}$                       (d)  $1-x$                       (e) None of these
15. Suppose that  $f(3-x) = 2x^2 - 5x + 4$  and  $f(x) = ax^2 + bx + c$ . What is  $a + b + c$ ?
- (a) -1                      (b) 0                      (c) 1                      (d) 2                      (e) 3
16. What is the remainder when  $7^{12}$  is divided by 12?
- (a) 1                      (b) 2                      (c) 3                      (d) 4                      (e) 6
17. What is the largest prime factor of  $3^{12} - 1$ ?
- (a) 13                      (b) 29                      (c) 47                      (d) 59                      (e) 73
18. How many real solutions are there to the equation  $|2-x| + |x-5| = 1$ ?
- (a) 4                      (b) 3                      (c) 2                      (d) 1                      (e) 0

19. Suppose that  $3 + 2\sqrt{2}$  solves  $x^2 - 6x + a = 0$ . Find the value of  $a$ .  
 (a) 1 (b) 3 (c) 5 (d) 6 (e) 8
20. Two numbers,  $x$  and  $y$  are selected at random from the interval  $[0, 3]$ . What is the probability that  $y \geq x + 1$ ?  
 (a)  $2/7$  (b)  $1/7$  (c)  $1/3$  (d)  $4/9$  (e)  $2/9$
21. Consider the sequence  $a_n$  defined by  $a_1 = 1, a_2 = 1 - 2, a_3 = 1 - 2 + 3, \dots, a_n = 1 - 2 + 3 - \dots + (-1)^{n+1}n$ . Find

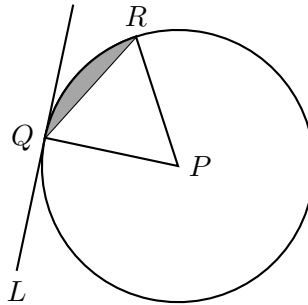
$$\sum_{n=1}^{99} a_n.$$

- (a) 99 (b) 100 (c) 50 (d) 49 (e) 0
22.  $\sqrt{7 - \sqrt{40}}$  equals  
 (a)  $\sqrt{7} - \sqrt{2}\sqrt[4]{10}$  (b)  $5\sqrt{10}$  (c)  $\sqrt{2} - \sqrt{5}$   
 (d)  $\sqrt{5} - \sqrt{2}$  (e) None of these
23. If  $a$  and  $b$  are real numbers, then  $(a - b\sqrt{-3})^3$  is real if and only if  
 (a)  $a = b^2$  or  $b = 0$  (b)  $a = -b^2$  or  $b = 0$  (c)  $a = b$  and  $b = 0$   
 (d)  $a = 0$  (e)  $a^2 = b^2$  or  $b = 0$
24. If  $y = 2x + 1$  is tangent to the circle centered at the origin in an  $xy$ -plane. Find the radius of the circle.  
 (a) 1 (b)  $\frac{\sqrt{3}}{3}$  (c)  $\frac{\sqrt{5}}{5}$  (d)  $\sqrt{2}$  (e)  $\frac{\sqrt{2}}{2}$
25. Find the area of the given figure. (*Note: Figure not drawn to scale!*)



- (a)  $\frac{\sqrt{3}}{4}$  (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{\sqrt{3}+1}{4}$  (d)  $\frac{\sqrt{3}+1}{2}$  (e) None of these
26. How many numbers between 1 and 200 are multiples of 3 or 5 or 7?  
 (a) 107 (b) 108 (c) 109 (d) 110 (e) 111

27. Let  $a + b = 1$  and  $a^3 + b^3 = 4$ . Find  $a^2 + b^2$ .
- (a)  $-1$                       (b)  $3$                       (c)  $\sqrt{3}$                       (d)  $\sqrt{3}/2$                       (e) None of these
28. The largest integer that divides  $n^4 - n^2$  for all  $n$  is
- (a)  $1$                       (b)  $2$                       (c)  $6$                       (d)  $12$                       (e)  $24$
29.  $\frac{\sqrt{\frac{6}{5}} + \sqrt{\frac{5}{6}}}{\sqrt{\frac{6}{5}} - \sqrt{\frac{5}{6}}}$  equals:
- (a)  $11$                       (b)  $\frac{91}{11}$                       (c)  $10$                       (d)  $12$                       (e)  $\frac{120}{11}$
30. How many terms are there in the expansion of  $(x + y + z + 1)^6$  ?
- (a)  $28$                       (b)  $49$                       (c)  $84$                       (d)  $108$                       (e)  $126$
31. If  $(x^2 - 5x + 6)^3 - (x - 2)^3$  is factored using only real coefficients, then the number of factors ( $x^2 = x \times x$  has two factors) is:
- (a)  $2$                       (b)  $3$                       (c)  $4$                       (d)  $5$                       (e)  $7$
32. Consider the circle below which has center at  $P$  with radius 2. If the acute angle between  $\overline{QR}$  and line  $L$  is 30 degrees, and the angle between  $L$  and  $\overline{QP}$  is 90 degrees, find the area of the shaded region. (*Note: Figure not drawn to scale!*)



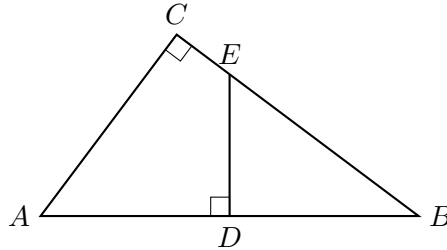
- (a)  $2\pi - \sqrt{3}$                       (b)  $\frac{2\pi - 3\sqrt{3}}{3}$                       (c)  $\frac{2\pi - 6\sqrt{3}}{3}$                       (d)  $\frac{4\pi - 3\sqrt{3}}{3}$                       (e) None of these
33. Let  $z$  be a solution of  $x^2 + x + 1 = 0$ . What is the value of  $z^{10} + z^5 + 1$ ?
- (a)  $0$                       (b)  $3$                       (c)  $1$                       (d)  $i\sqrt{3}$                       (e) None of these

34. Let  $n$  be the smallest positive integer such that the expansion of  $(x^2 + \frac{1}{x^3})^n$  has a constant term. Determine the constant term.
- (a) 10                      (b) 12                      (c) 15                      (d) 20                      (e) 21
35. Evaluate the sum:
- $$\cos 10^\circ + \cos 20^\circ + \cdots + \cos 170^\circ + \cos 180^\circ$$
- (a) 1                      (b) 0                      (c) 2                      (d) -1                      (e) None of these
36. How many positive integers  $d$  such that the remainder in dividing 4049 by  $d$  equals 24?
- (a) 6                      (b) 8                      (c) 11                      (d) 16                      (e) None of these
37. Grade  $A$  oil sells for \$78 per unit and grade  $B$  oil sells for \$54 per unit. If a mixture sells for \$63 per unit, find the ratio of  $A$  to  $B$  used in the mixture.
- (a)  $\frac{11}{15}$                       (b)  $\frac{2}{3}$                       (c)  $\frac{3}{5}$                       (d)  $\frac{3}{4}$                       (e) None of these
38. Find the sum of  $\frac{1}{10 \cdot 11} + \frac{1}{11 \cdot 12} + \cdots + \frac{1}{19 \cdot 20}$ .
- (a)  $\frac{1}{10}$                       (b)  $\frac{1}{11}$                       (c)  $\frac{1}{19}$                       (d)  $\frac{1}{20}$                       (e) None of these
39. If a ball 2 inches in diameter weighs 32 ounces, then what is the weight of the ball 3 inches in diameter made of the same material?
- (a) 72                      (b)  $32\pi$                       (c) 108                      (d) 124                      (e) None of these
40. If  $x$  is a solution to  $9^x - 9^{x-1} = 24$ , then  $x$  satisfies:
- (a)  $-1 < x < 0$                       (b)  $0 < x < 1$                       (c)  $1 < x < 2$                       (d)  $2 < x < 3$                       (e) None of these
41. Let  $1 + i$  be a root of  $x^3 + ax^2 + bx - 12 = 0$  where  $a$  and  $b$  are real numbers. Find  $a + b$ ?
- (a) -5                      (b) 5                      (c) -6                      (d) 6                      (e) 0
42. What is the remainder when  $4^{10} + 6^{10}$  is divided by 25?
- (a) 12                      (b) 8                      (c) 4                      (d) 2                      (e) 1
43. Among the 10 numbered cards, from 1 to 10, we will pick 5 cards randomly. What is the probability that number 7 card is excluded from the selection?
- (a)  $\frac{1}{10}$                       (b)  $\frac{1}{5}$                       (c)  $\frac{1}{2}$                       (d)  $\frac{2}{5}$                       (e) None of these

44. The length of an interior diagonal of a rectangular box is 8 cm, and the sum of the lengths of all the edges of the box is 36 cm. Find the surface area of the box.

(a)  $12 \text{ cm}^2$       (b)  $14 \text{ cm}^2$       (c)  $16 \text{ cm}^2$       (d)  $17 \text{ cm}^2$       (e)  $19 \text{ cm}^2$

45. In the figure,  $AB = 20$ ,  $AC = 12$ ,  $AD = DB$ , angles  $ACB$  and  $ADE$  are right angles. Find the area of the quadrilateral  $ADEC$ . (Note: Figure not drawn to scale!)



(a) 58.5      (b) 61      (c) 49.5      (d) 62.5      (e) None of these

46. Which one of the following is equal to  $\cos \frac{\pi}{5}$  ?

(a)  $\frac{-1+\sqrt{5}}{4}$       (b)  $\frac{1+\sqrt{5}}{4}$       (c)  $\frac{\sqrt{2}+\sqrt{3}}{4}$       (d)  $\frac{-\sqrt{2}+\sqrt{3}}{4}$       (e) None of these

47. The function  $g(x) = f(5+x) + f(-1-x)$  is symmetric about the vertical line  $x = a$  for all functions  $f$  defined on the reals. Find the value  $a$ .

(a)  $-3$       (b)  $-2$       (c)  $0$       (d)  $1$       (e)  $2$

48. A sequence begins with  $a_1, a_2$ , and for  $n > 2$  is defined by  $a_n = a_{n-1} - a_{n-2}$ . Find  $a_{2018}$ .

(a)  $a_1$       (b)  $a_2$       (c)  $a_1 - a_2$       (d)  $a_2 - a_1$       (e)  $-a_2$

49. Which one of the following is equal to

$$\sqrt[3]{7 + 5\sqrt{2}} + \sqrt[3]{7 - 5\sqrt{2}}.$$

(a)  $2\sqrt[3]{3}$       (b)  $2$       (c)  $\sqrt{5}$       (d)  $2\sqrt{5}$       (e) None of these

50. The increasing sequence of positive integers  $a_1, a_2, a_3, \dots$  has the property that

$$a_{n+2} = a_{n+1} + a_n \text{ for all } n \geq 1.$$

If  $a_6 = 49$ , what is  $a_3$ ?

(a) 3      (b) 5      (c) 8      (d) 11      (e) 19

51. How many positive factors of  $172^3 - 89^3 - 83^3$  are there?

(a) 8      (b) 16      (c) 24      (d) 26      (e) 48

52. In a certain village live 27 families. Each family has one, two, or three cars. There are as many families owning three cars as families with only one. How many cars are there in the village?

53. For how many number of real numbers  $x$  is  $(x + i)^5$  real?

54. Given that the one roots of the equation  $x^2 - 2ax + m = 0$  is  $a - b$ , determine  $m$  in terms of  $a$  and  $b$ . (Completely expand the result.)

55. Find the set of all real numbers  $x$  for which

$$2x + \sqrt{4x^2 + 1} - \frac{1}{2x + \sqrt{4x^2 + 1}}$$

is a rational number.

56. Recall that for real number  $x$ ,  $[x]$  denotes the greatest integer not exceeding  $x$ . Find all real number pairs  $(x, y)$  satisfying the following equations:

$$x + y - [y] = 30.1$$

$$x + [x] + [y] = 71.7$$

57. Compute

$$\sqrt{(19)(20)(21)(22) + 1}.$$

▷ KEYS ◁

[1] (a)	[16] (a)	[31] (d)	[46] (b)
[2] (d)	[17] (e)	[32] (b)	[47] (a)
[3] (b)	[18] (e)	[33] (a)	[48] (b)
[4] (b)	[19] (a)	[34] (a)	[49] (b)
[5] (a)	[20] (e)	[35] (d)	[50] (d)
[6] (a)	[21] (c)	[36] (b)	[51] (e)
[7] (d)	[22] (d)	[37] (c)	[52] 54
[8] (a)	[23] (e)	[38] (d)	[53] 4
[9] (b)	[24] (c)	[39] (c)	[54] $a^2 - b^2$
[10] (e)	[25] (a)	[40] (c)	[55] The set of all rational numbers.
[11] (b)	[26] (d)	[41] (d)	[56] (29.7, 13.4)
[12] (a)	[27] (b)	[42] (d)	[57] 419
[13] (d)	[28] (d)	[43] (c)	
[14] (c)	[29] (a)	[44] (d)	
[15] (d)	[30] (c)	[45] (a)	