NMC Sample Problems: Grade 11

1. Which one of the following functions has different domain?

(a) $\frac{1}{x}$ (b) $\frac{1}{x^{10}}$ (c) $\frac{1}{x^{11}+1}$ (d) $\frac{4}{x\sqrt{x^2+1}}$ (e) $\frac{x-1}{x(x^2+1)}$

2. In the expansion of $(x+y)^8$, what is the coefficient of x^4y^4?

(a) 60 (b) 70 (c) 80 (d) 90 (e) 100

3. Find the equation of the circle centered at $(\frac{1}{2}, 0)$ passing through the point $(0, 1)$.

(a) $(x-\frac{1}{2})^2+y^2=\frac{5}{4}$ (b) $(x+\frac{1}{2})^2+y^2=\frac{\sqrt{5}}{2}$

(c) $x^2+(y-\frac{1}{2})^2=\frac{\sqrt{5}}{2}$ (d) $(x-\frac{1}{2})^2+y^2=\frac{\sqrt{5}}{2}$

(e) $x^2+(y-\frac{1}{2})^2=\frac{5}{4}$

4. What is the solution set of the following equation: $\ln(x^2-x)=\ln(2x-2)$?(Here, $\ln x=\log_e x$)

(a) $\{0\}$ (b) $\{1\}$ (c) $\{2\}$ (d) $\{1,2\}$ (e) $\{0,1\}$

5. Evaluate $\log_{1/2} 8$.

(a) -3 (b) -2 (c) -1 (d) 1 (e) 2

6. Find the sum of the maximum and minimum values of the expression $-10\sin(x+2)+5$.

(a) -5 (b) 5 (c) -10 (d) 10 (e) 15

7. Find the sum of all solutions of the equation $|2x-1|-|x-5|=100$.

(a) 8 (b) -8 (c) 3 (d) -3 (e) 0

8. Find the polar coordinates (r,θ) for the point with rectangular coordinates $(x,y)=(\sqrt{2},\sqrt{2})$.

(a) $(2,\frac{\pi}{4})$ (b) $(\sqrt{2},\frac{\pi}{4})$ (c) $(2,\pi)$ (d) $(2,\frac{3\pi}{2})$ (e) $(2,2\pi)$

9. If $10^x=\frac{1}{2}$, then which of the following is 10^{2x-1}?

(a) 1 (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{40}$ (e) $\frac{3}{40}$

10. If $\csc x=10$ and $-90^\circ<x<90^\circ$, then what is $\cos x$?

(a) $\frac{3\sqrt{11}}{5}$ (b) $-\frac{3\sqrt{11}}{5}$ (c) $\frac{3\sqrt{11}}{10}$ (d) $-\frac{3\sqrt{11}}{10}$ (e) $\frac{\sqrt{11}}{10}$

11. Find the equation of the straight line which passes through the point $(0, 0)$ and is perpendicular to $83x + 24y + 1 = 0$.

(a) $83x - 24y = 0$ (b) $24x + 83y = 0$ (c) $24x - 83y = 0$

(d) $24x - 83y + 1 = 0$ (e) $24x + 83y - 1 = 0$

12. Find the vertex point of the parabola $x - y^2 + 1 = 0$.

(a) $(1, 0)$ (b) $(-1, 0)$ (c) $(0, 1)$ (d) $(0, -1)$ (e) $(-1, -1)$

13. Find the rectangular coordinates (x, y) for the point with polar coordinates $(r, \theta) = (2, \frac{3\pi}{4})$.

(a) $(-\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, -\sqrt{2})$ (c) $(-\sqrt{2}, -\sqrt{2})$ (d) $(\sqrt{2}, \sqrt{2})$ (e) $(2, 2)$

14. When $\cos\theta = \frac{1}{5}$ and $\tan\theta > 0$, find $\sin\theta$.

(a) $\frac{4}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{2\sqrt{6}}{5}$ (d) $-\frac{2\sqrt{6}}{5}$ (e) $\frac{3}{5}$

15. Let x and y be real numbers satisfying the following two conditions:

$$x - y = 1 \quad \text{and} \quad x^3 - y^3 = 16$$

Find $x^2 + y^2$.

(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

16. Find the sum $1 + 10 + 10^2 + \cdots + 10^{10}$.

(a) $\frac{10^{10}-1}{9}$ (b) $\frac{10^{10}+1}{9}$ (c) $\frac{10^{10}}{9}$ (d) $\frac{10^{11}+1}{9}$ (e) $\frac{10^{11}-1}{9}$

17. What is the remainder when $1! + 2! + 3! + \cdots + 2018!$ is divided by 10?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 0

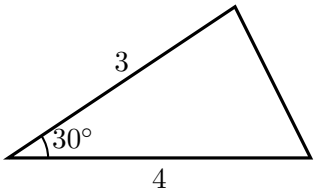
18. How many seven digit numbers contain the digit pattern "2018" exactly once?

(a) 2700 (b) 3000 (c) 3400 (d) 3700 (e) 4000

19. Let α, β and γ be three roots of $x^3 + 1 = 0$. Find the product $p(\alpha)p(\beta)p(\gamma)$, where $p(x) = x^4 + 4x^3 - x + 4$.

(a) -8 (b) -4 (c) 0 (d) 4 (e) 8

20. Find the area of the following triangle.



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

21. Suppose a, b, c and d are integers, and $x = \sqrt{1+\sqrt{2}}$ is a solution of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. What is the sum $a + b + c + d$?

(a) 3 (b) -3 (c) 6 (d) -6 (e) -15

22. What is the remainder when $x^{2020} + 2020$ is divided by $x + 1$?

(a) 2017 (b) 2018 (c) 2019 (d) 2020 (e) 2021

23. Find the value of the product:

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{100^2}\right)$$

(a) $\frac{101}{100}$ (b) $\frac{101}{200}$ (c) $\frac{101}{300}$ (d) $\frac{101}{400}$ (e) $\frac{101}{500}$

24. Which one is the number of the real solutions of the equation $e^{-x} - \sin x = 0$?

(a) 0 (no solutions) (b) 2 (two solutions) (c) 4 (four solutions)

(d) 6 (six solutions) (e) ∞ (infinitely many solutions)

25. A ball is thrown vertically upward and its height after t seconds is $h(t) = 6.5 + 40t - 32t^2$ feet. Find the maximum height reached by the ball.

(a) 16 feet (b) 17 feet (c) 18 feet (d) 19 feet (e) 20 feet

26. The function $f(x) = \frac{x}{x-1}$ has one horizontal asymptote, say $y = a$, and one vertical asymptote, say $x = b$. Find $a + b$.

(a) -1 (b) 0 (c) 1 (d) 2 (e) 3

27. If $\sin x + \cos x = \frac{9}{7}$, then what is the value of $|\sin x - \cos x|$?

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{17}}{7}$ (d) $\frac{1}{8}$ (e) $\frac{2}{9}$

28. Find the remainder when $1 + 2 + 3 + \cdots + 1999 + 2000$ is divided by 1000.

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

29. Evaluate $\left(\dfrac{-1+\sqrt{3}i}{2}\right)^{100}$.

(a) 0 (b) 1 (c) $\frac{-1+\sqrt{3}i}{2}$ (d) $\frac{-1-\sqrt{3}i}{2}$ (e) $\frac{1+i}{2}$

30. Let $a_{100}, a_{99}, \cdots, a_1$ and a_0 be the coefficients of the expansion of the expression $(x+1)^{100}$, that is

$$(x+1)^{100} = a_{100}x^{100} + a_{99}x^{99} + \cdots + a_1x + a_0$$

Find the sum of even coefficients, $a_{100} + a_{98} + \cdots + a_2 + a_0$.

(a) 2^{99} (b) 2^{98} (c) 2^{97} (d) 2^{96} (e) 2^{95}

31. Find the minimum value of $3\sin x - 4\cos x + 1$.

(a) 4 (b) 0 (c) -4 (d) -8 (e) -12

32. Find the solution set of the equation $\left(5^{2x+1}\right)^{x+8} = 5^2$.

(a) $\{\frac{-2\pm\sqrt{29}}{7}\}$ (b) $\{\frac{-21\pm\sqrt{29}}{7}\}$ (c) $\{\frac{19\pm\sqrt{222}}{7}\}$ (d) $\{\frac{-17\pm\sqrt{241}}{4}\}$ (e) $\{\}$ (No solution)

33. If $\sin x + \cos x = \frac{1}{2}$, then what is the value of $\sin^3 x + \cos^3 x$?

(a) $-\frac{11}{16}$ (b) $\frac{11}{16}$ (c) $-\frac{3}{8}$ (d) $\frac{3}{8}$ (e) $\frac{1}{8}$

34. There are 2 red ball, 2 blue balls, and 4 white balls in a bag. If James takes out four balls at once, then what is the probability that he takes out at least two white balls?

(a) $\frac{17}{70}$ (b) $\frac{53}{70}$ (c) $\frac{1}{7}$ (d) $\frac{6}{35}$ (e) $\frac{1}{5}$

35. A committee composed of Alice, Mark, Ben, Connie and Francisco is about to select two representatives randomly. What is the probability that Ben is not included in the selection?

(a) $\frac{3}{5}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$ (e) $\frac{2}{5}$

36. Find the sum of all integers between 1 to 1000 (inclusive) that are not multiples of 8.

(a) 500×875 (b) 400×875 (c) 300×875 (d) 200×875 (e) 100×875

37. Evaluate the following($\ln x = \log_e x$):

$$\ln\left(\frac{1}{2}\right) + \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3}{4}\right) + \cdots + \ln\left(\frac{999}{1000}\right)$$

(a) $3\ln 5$ (b) $-3\ln 5$ (c) $3\ln 10$ (d) $-3\ln 10$ (e) 10

38. How many functions which are one-to-one are there from the set $X = \{a, b, c\}$ to the set $Y = \{1, 2, 3, 4\}$?

(a) 10 (b) 24 (c) 30 (d) 40 (e) 64

39. Let $z = a + i$ be a complex number, where a is a positive real number ($i^2 = -1$). If the real part of z^2 equals that of $\bar{z}$, then what is a?

(a) 1 (b) $\frac{1+\sqrt{5}}{2}$ (c) $\frac{-1+\sqrt{5}}{2}$ (d) $\frac{2+\sqrt{5}}{2}$ (e) 2

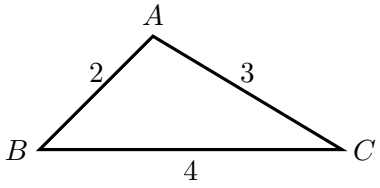
40. What is the average of the numbers in the set $\{-7, -2, 3, 8, \cdots, 93\}$ given by an arithmetic progression?

(a) 41 (b) 42 (c) 43 (d) 44 (e) 45

41. The rational function $f(x) = \frac{ax+b}{x}$ has the inverse function $f^{-1}(x) = \frac{3}{2x+1}$. Find $a + b$.

(a) 1 (b) -1 (c) 0 (d) 2 (e) -2

42. Find $4\cos B + 3\cos A$ of the following triangle.



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

43. An employee of a computer store is paid a base salary \$2,535 a month plus a 5% commission on all sales over \$4,000 during the month (For example, if the gross sales during the month is \$5000, then the commission is \$50). How much must the employee sell in a month to earn a total of \$2,852 for the month?

(a) \$6,200 (b) \$7,200 (c) \$9,200 (d) \$10,340 (e) \$20,340

44. A speedboat takes a half hour longer to go 8 miles up a river than to return. If the boat cruises at 20 miles per hour in still water, what is the rate of the current? (The units are miles/hour.)

(a) $4(\sqrt{41} - 4)$ (b) $4\sqrt{2}$ (c) $4(\sqrt{2} - 1)$ (d) 6 (e) 8

45. Which one of the following is the greatest?

(a) 80^5 (b) 63^5 (c) 8^{10} (d) 6^{10} (e) 3^{20}

46. Suppose that a, b are integers and $1 - \sqrt{5}$ is a solution of the equation $x^3 + x^2 + ax + b = 0$. Find $a + b$.

(a) 10 (b) -10 (c) 22 (d) -22 (e) 0

47. Harry performed the calculation: $5 \times 8 = 44$. It turns out Harry's calculation is correct in base b. Find the value b.

(a) 9 (b) 10 (c) 11 (d) 12 (e) 13

48. Suppose that $f(x)$ is a polynomial with integer coefficients, having 100 and 200 as zeros. Which of the following could possibly be the value of $f(150)$?

(a) 2 (b) $2^2 \cdot 5$ (c) 5^2 (d) $2 \cdot 5^4$ (e) $2^3 \cdot 5^4$

49. A polynomial $f(x) = x^3 + ax^2 + bx + c$ satisfies $f(1) = 1$, $f(2) = 2$ and $f(3) = 3$. What is the remainder when $f(x)$ is divided by $x - 4$?

(a) 10 (b) 5 (c) 0 (d) -5 (e) -10

50. Evaluate $\dfrac{2020^4 + 3 \cdot 2020^3 - 3 \cdot 2020^2 + 6 \cdot 2020 + 8}{2020^3 - 2020^2 + 2020 + 2}$.

(a) 2016 (b) 2018 (c) 2020 (d) 2022 (e) 2024

51. The domain of the function

$$f(x) = \log_{10}(\log_2(\log_3(\log_5 x)))$$

is $\{x | x > c\}$. What is the value of c?

(a) 0 (b) 1 (c) 10 (d) 100 (e) 125

52. What is the solution set of the following equation: $\ln|x^2 - 2x + 1| = \ln|3x - 3|$? (Here, $\ln x = \log_e x$)

(a) $\{0\}$ (b) $\{1\}$ (c) $\{1, 4, -2\}$ (d) $\{4, -2\}$ (e) $\{0, 1\}$

53. What is the number of real solutions of the equation $|\sin x| - \frac{2}{\pi}|x| = 0$?

(a) 1 (one solutions) (b) 2 (two solutions) (c) 3 (three solutions)

(d) 4 (four solutions) (e) ∞ (infinitely many solutions)

54. Let $a_{2020}, a_{2019}, \cdots, a_1$ and a_0 be the coefficients of the expansion of the expression $(x+2)^{2020}$, that is

$$(x+2)^{2020} = a_{2020}x^{2020} + a_{2019}x^{2019} + \cdots + a_1 x + a_0$$

What is the sum of even coefficients $a_{2020} + a_{2018} + a_{2016} + a_{2016} + \cdots + a_2 + a_0$ of all even indices.

(a) 0 (b) 1 (c) -1 (d) $\frac{1+3^{2020}}{2}$ (e) $\frac{1-3^{2020}}{2}$

55. There are 3 red ball, 2 blue balls, and 4 white balls in a bag. If James takes out five balls at once, then what is the probability that he takes out at least three white balls?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{11}{14}$ (e) $\frac{45}{126}$

56. Suppose that a, b are integers and $2+\sqrt{3}$ is a solution of the equation $x^3 + ax^2 + bx + 1 = 0$. Find $a + b$.

(a) 0 (b) -5 (c) 5 (d) -6 (e) 6

57. A polynomial $f(x)$ of degree ≥ 2 whose coefficients are all integers satisfies $f(2) = 2$ and $f(-2) = -2$. What is the remainder when $f(x)$ is divided by $x^2 - 4$?

58. Let f be a function satisfying the following equation:

$$f(x) = f(x+1) \text{ for all real numbers } x$$

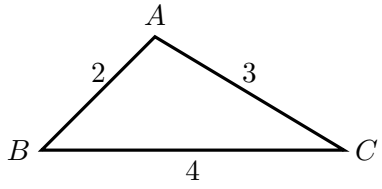
Suppose further $f(x) > 0$ for all x. What is the value of $\frac{f(\pi+1)}{f(\pi-1)}$?

59. The sides a, b, c of a right triangle satisfy the following equation

$$a^2 + b^2 + c^2 = 1.$$

Find the length of the hypotenuse?

60. Find $\frac{\sin A + \sin B + \sin C}{\cos A + \cos B + \cos C}$ of the following triangle.



61. Find the remainder when 1234^{4321} is divided by 5.

62. If $\log_9 4 = a$, then what is $\log_{16} 81$ in terms of a?

63. Let a and b be integers which satisfy $\sqrt{43 + 30\sqrt{2}} = a + b\sqrt{2}$. Find $a + b$.

64. Let f be a function satisfying the following equation:

$$f(x) = \pi x f(1-x) \text{ for all real numbers } x.$$

Find $f(100)$.

65. How many ordered pairs (x, y) of real numbers are there which satisfy the following equations?

$$x^{100} + y^{100} = x^{101} + y^{101} = x^{102} + y^{102}.$$

66. The sides of a right triangle form an arithmetic progression. Find the ratio of the shorter leg to the hypotenuse.

▷ KEYS ◁

[1] (c)

[2] (b)

[3] (a)

[4] (c)

[5] (a)

[6] (d)

[7] (b)

[8] (a)

[9] (d)

[10] (c)

[11] (c)

[12] (b)

[13] (a)

[14] (c)

[15] (c)

[16] (e)

[17] (a)

[18] (d)

[19] (e)

[20] (c)

[21] (b)

[22] (e)

[23] (b)

[24] (e)

[25] (d)

[26] (d)

[27] (c)

[28] (e)

[29] (c)

[30] (a)

[31] (c)

[32] (d)

[33] (b)

[34] (b)

[35] (a)

[36] (a)

[37] (d)

[38] (b)

[39] (b)

[40] (c)

[41] (a)

[42] (b)

[43] (d)

[44] (a)

[45] (e)

[46] (d)

[47] (a)

[48] (e)

[49] (a)

[50] (e)

[51] (e)

[52] (d)

[53] (c)

[54] (d)

[55] (e)

[56] (d)

[57] x

[58] 1

[59] $\frac{1}{\sqrt{2}}$

[60] $\frac{3\sqrt{15}}{7}$

[61] 4

[62] $\frac{1}{a}$

[63] 8

[64] 0

[65] 4

[66] 3/5