Sample Test Group B
Grade 6 ~ Grade 8 Part 1

1. A cup of coffee at a coffee shop costs \$3 and muffins cost \$4 each. How much will it cost to buy 15 cups of coffee and 9 muffins?

(A) \$12 **(B)** \$81 **(C)** \$87 **(D)** \$100 **(E)** \$135

Answer: (B)

2. Define an operation $\bigotimes$ as follows:

$$x \bigotimes y = x \cdot y - 3x$$

for all integers x and y. Then $2 \bigotimes 1 = 2 \cdot 1 - 3 \cdot 2 = -4$. What is the value of $(3 \bigotimes 5) \bigotimes 2$?

(A) 24 **(B)** -4 **(C)** -6 **(D)** -8 **(E)** -24

Answer: (C)

3. There are 48 students on a school bus. Thirty one students wear hats. Twenty three students wear eye glasses. Nine students wear neither eye glasses nor hats. How many students wear both hats and eye glasses?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

Answer: (D)

4. The ratio of teachers to students in a particular school is 5 to 26. The ratio of female students to the total number of students is 6 to 13. If there are 72 female students, how many teachers are there?

(A) 11 **(B)** 12 **(C)** 20 **(D)** 25 **(E)** 30

Answer: (E)

5. Suppose $\frac{x}{y} = \frac{7}{9}$ and $\frac{y}{z} = \frac{1}{4}$. What is the value of $\frac{x+y}{z}$?

(A) $\frac{4}{9}$ **(B)** $\frac{7}{4}$ **(C)** $\frac{7}{36}$ **(D)** $\frac{1}{2}$ **(E)** 2

Answer: (A)

6. Which number is the largest in the set $\{3^{475}, 4^{380}, 5^{285}, 6^{190}, 7^{95}\}$?

(A) 3^{475} **(B)** 4^{380} **(C)** 5^{285} **(D)** 6^{190} **(E)** 7^{95}

Answer: (B)

7. Joe was shopping for shoes and found a pair that was on sale for 25%. As he went to pay for it, the store manager announced an instant sale that took an additional 10% off all items. What percent would be the total discount?

(A) 2.5% **(B)** 25% **(C)** 32.5% **(D)** 35% **(E)** 40%

Answer: (C)

8. Find the value of

$$2021\frac{1}{2} - 2 + 3 \div \frac{3}{4} - 2010\frac{2}{3}.$$

(A) $12\frac{2}{3}$ **(B)** $12\frac{5}{6}$ **(C)** $13\frac{1}{6}$ **(D)** $13\frac{2}{3}$ **(E)** $13\frac{5}{6}$

Answer: (B)

9. There are three lines determined by $x + 2y = 3$, $4x + 5y = 6$, and the x-axis. Find the area of the triangle that is bounded by all the three lines.

(A) 1.8 **(B)** 1.75 **(C)** 1.725 **(D)** 1.5 **(E)** 1.125

Answer: (D)

10. Ella collects dimes, quarters, half dollars, and $1 coins in her piggy bank. When she opens it, the number of dimes equals the number of half dollar coins. The number of quarters also equals the number of $1 coins. Knowing that there is at least one of each coin in the piggy bank and that the value of the coin collection is between $10 and $12, find the number of unique possible combinations of coins in the collection.

(A) 15 **(B)** 16 **(C)** 21 **(D)** 24 **(E)** 25

Answer: (E)

11. A positive integer from 1 to 1000 is randomly selected. Find the probability that the selected number is a multiple of either 2 or 3, but not a multiple of 9.

(A) $\frac{139}{250}$ **(B)** $\frac{143}{250}$ **(C)** $\frac{267}{500}$ **(D)** $\frac{361}{500}$ **(E)** $\frac{667}{1000}$

Answer: (A)

12. If $x < 0$, which of the following options is the third largest?

(A) $x^2 - x$ **(B)** $x + x^3$ **(C)** $x^2 - x^3$ **(D)** $x^3 - x^4$ **(E)** x^3

Answer: (E)

13. There is a tank with 100 L of water where 4 kg of salt is dissolved. You open a faucet to add a salt solution of 0.6 kg/L at the constant speed of 10 L/min. When do you have to close the faucet if you want the concentration of the salt solution in the tank to be 0.25 kg/L in the tank? Find the time it takes after the faucet is open to the nearest minute.

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

Answer: (D)

14. If $20^{21} = p^a q^b$ where both p and q are prime, what is $p + q + a + b$?

(A) 49 **(B)** 55 **(C)** 63 **(D)** 68 **(E)** 70

Answer: (E)

15. BTS randomly selects a day in the month of November to release the group's new album. What is the probability that the album is released on a prime number date?

(A) $\frac{1}{3}$ **(B)** $\frac{11}{30}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{10}$ **(E)** $\frac{5}{15}$

Answer: (A)

16. If the three sides of a right triangle form an arithmetic sequence, and the perimeter of the triangle is 60, what is the length of the hypotenuse? (Note that an *arithmetic sequence* is a sequence of numbers such that the difference between the consecutive terms is constant.)

(A) 23 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

Answer: (C)

17. Find the area of the square inscribed in a half circle with radius 5.

(A) 24 **(B)** 20 **(C)** 15 **(D)** 10 **(E)** 5

Answer: (B)

18. How many positive integers less than 1000 have exactly 3 divisors?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

Answer: (B)

19. How many ordered triples of positive integers (a, b, c) satisfy

$$(a^b)^c = 16?$$

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Answer: (E)

20. Suppose that

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots - \frac{1}{10} + \frac{1}{11} = \frac{p}{q},$$

where p and q are relatively prime integers. Which of the following is a divisor of p?

(A) 5 **(B)** 7 **(C)** 13 **(D)** 17 **(E)** 19

Answer: (D)

Grade 6 Part 2

1. A straight line passes through the three points $(8, 4)$, $(12, 7)$ and $(x, 1)$. What is the value of x?

Answer: 4

2. For whole numbers between 1 and 1001, inclusive, let m be the sum of odd numbers and n be the sum of even numbers. What is the value of $m - n$?

Answer: 501

3. Find the number of pairs of integers, (x, y), satisfying the inequality, $9 < x^2 + y^2 < 49$.

Answer: 116

4. There are two water pumps available to fill a pool. One pump can fill a pool three times faster than the other pump. When both pumps are used, they fill the pool in six hours. How many hours would it take to fill the pool if only the faster pump is used?

Answer: 8 hours

5. The following is called Pascal's Triangle.

0								1						
1							1		1					
2						1		2		1				
3					**1**		**3**		**3**		**1**			
4				1		4		6		4		1		
5			1		5		10		10		5		1	
6		1		6		15		20		15		6		1
⋮								⋮						

It is known that Pascal's Triangle can show the coefficients in Binomial expansion, $(x+y)^n$. For example, when $n = 3$, $(x + y)^{\mathbf{3}} = \mathbf{1} \cdot x^3 + \mathbf{3} \cdot x^2y + \mathbf{3} \cdot xy^2 + \mathbf{1} \cdot y^3$.

Using this pattern, simplify the following expression

$$(x + y)^{2021} + (x - y)^{2021}$$

by expanding it and combining like terms. How many distinct terms are in the simplified expression?

Answer: 1011

Grade 7 Part 2

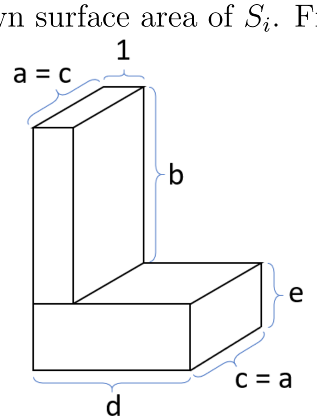
1. Let $a^2 - b^2 = 2021$, where a and b are positive integers. Find the smallest possible value of ab.

Answer: 90

2. Yesterday, Ben started hiking at 7:00 AM and finished a one-way hike in 140 minutes, travelling at a constant speed. When he was three fifths of the way done with the hike, he encountered a runner on the same trail who started running k minutes later than Ben at a constant speed that was four thirds of Ben's speed. Assume that all speeds and the runner's starting time remain constant. Let 6:xy AM be the latest time at which Ben should begin hiking on the trail in order to avoid the runner, including at the finish line. For example, if Ben starts at 6:01 AM and meets the runner at the finish line, then he should begin hiking at 6:00 AM. Find $k + y$. Note that xy represents a two-digit number where x is the tens digit and y is the units digit.

Answer: 26

3. Amy constructs one backrest chair with a solid rectangular seat but no armrests using 27 unit cubes. She needs to use all the cubes for a chair. The height of the backrest measured from the top of the seat's base needs to be larger than or equal to one and a half times the seat height. The dimensions of the backrest are $a \times b \times 1$, where neither a nor b is 1. The dimensions of the seat viewed from its bottom are $c \times d \times e$, where none of c, d, or e is 1. With all the restrictions above she can construct k different chairs. Each chair, C_i has its own surface area of S_i. Find $k + S_1 + S_2 + \cdots + S_k$.



Answer: 140

4. Let $x^2 - 3xy + 21y = 7x + c$, where c is a single-digit prime number. Allowing only integer coordinates for (x, y), there exist k solutions for the equation. Find $(x_1^2 + y_1^2) + (x_2^2 + y_2^2) + \cdots + (x_k^2 + y_k^2)$.

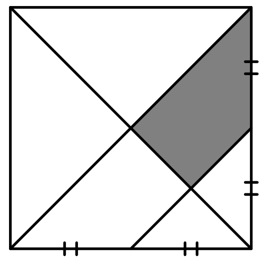
Answer: 321

5. Given a square $ABCD$, let CE be an extension of CD. Let F be the intersection of AD and BE on AD. If the ratio of the area of ABF to the area of $BCDF$ is $\frac{2}{3}$, the ratio of CD to DE in the simplest form is $\frac{k}{l}$. Find $k^2 + l$.

Answer: 17

Grade 8 Part 2

1. A square with one side length 12 is divided, as shown. What is the area of the shaded region?



Answer: 27

2. Alicia's birthday is on January 15, and the birthdays of her two siblings, Brett and Clair, come later in the year in that order. One day, Alicia calculated the number of days between two neighboring birthdays in a non-leap year cycle: from Alicia's birthday to Brett's birthday, Brett's birthday to Clair's birthday, and Clair's birthday to Alicia's next birthday. (Alicia counted the number of days by excluding the first day and including the last day. For example, there are 12 days from January 15 to January 27.) Then she learned that the three numbers form an increasing geometric sequence. How many days are there from Clair's birthday to Alicia's birthday?

Answer: 320

3. Suppose that

$$\sqrt{\left(\sqrt[3]{2}+1\right)^2+3}=\sqrt[3]{a}+\sqrt[3]{b},$$

where a and b are positive integers. What is $a+b$?

Answer: 6

4. Yuna rolls three six-sided standard dice and multiplies the three numbers on the top faces. If the probability of getting the product a perfect square is equal to $\frac{p}{q}$ where p and q are relatively prime integers, what is $p+q$?

Answer: 127

5. Let a_n be a sequence defined by

$$a_n = n^2 + 1.$$

Then the product of four consecutive terms in a_n can be written as a product of two terms in a_n. Find $p+q$ if

$$a_{11}a_{12}a_{13}a_{14} = a_p a_q.$$

Answer: 316